MATH 54 21 April 2014

FIRST LONG EXAM Summer; AY 2013-2014

General Instructions: Answers written using anything other than black or blue ballpen may not be corrected. Items with insufficient or disorganized solutions may not gain full points. Any form of cheating or academic dishonesty is subject to disciplinary action.

I. Solve the integral $\displaystyle\int \frac{1}{x^4\sqrt{x^2-1}}\,dx$.

II. Solve the integral $\displaystyle\int \frac{2x^5 - x^4 + 20x^3 + x^2 + 9}{x^4 + 9x^2}\,dx$.

III. Solve the improper integral $\displaystyle\int_0^\infty \sqrt{x}e^{-\sqrt{x}}\,dx$. (Hint: Use a substitution first.)

IV. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x\ln x}{\cos^2(3y)}, \qquad y(1) = \frac{\pi}{6}.$$

V. Find the orthogonal trajectory to the family of curves satisfying the equation $y - 1 = e^{x^2-x+k}$. (No need to isolate y in the answer.)

VI. The following differential equation gives us a model of a zombie outbreak in a town with an initial population of 200:

$$\frac{dz}{dt} = k \cdot z \cdot (200 - z)$$

where z is the number of zombies in the town, t is the time measured in hours and k is a constant. Aaron, a citizen of the town of 200, suddenly becomes a zombie at 12:00 am. At 1:00 am (one hour after), it was observed that 4 people (still including Aaron) have become zombies.

1. Solve for an equation relating z and t. (Hint: Solve for k along the way.)
2. How many hours after 12:00 am will the number of zombies reach 50? (You may express your answer in terms of ln).

END OF EXAM.

"Only once, you live." –Yoda